1(a)   Without tables, simplify

$$\frac{2\frac{1}{5} \times 5\frac{5}{6}}{6\frac{5}{12}}$$

(b)   Evaluate $\log_3 81 + \log_3 27 - \log_3 243$

Solution

$$\frac{2\frac{1}{5} \times 5\frac{5}{6}}{6\frac{5}{12}}$$

Simplifying the denominator and the numerator, we have

$$= \frac{\frac{11}{5} \times \frac{35}{6}}{\frac{77}{2}} = \frac{11}{5} \times \frac{35}{6} \div \frac{77}{12}$$

$$= \frac{11}{5} \times \frac{35}{6} \times \frac{12}{77}$$

$$= 2$$

1(b) $\log_3 81 + \log_3 27 - \log_3 243$ using the similar root power property of the logarithm,

we have $\log_3\left(\frac{81 \times 27}{243}\right) = \log_3\left(\frac{3^4 \times 3^3}{3^5}\right) = \log_3\left(\frac{3^7}{3^5}\right) = \log_3 9,$

$$= \log_3 3^2 = 2\log_3 3 = 2$$

2(a)   Ade, Eyo and Nuhu contributed ₦50,000, ₦75,000 and ₦100,000 respectively to a joint business venture. Their profit is to be shared in the ratio of their contributions. If they made a profit of ₦135,000 how much will each receive?

(b)   Solve the following linear equation: $10(3x - 2) = 7(5x - 4)$

Solution

(a)   Since Ade, Eyo and Nuhu contributed ₦50,000, ₦75,000 and ₦100,000, the ratio of their contribution will be 2:3:4
Since Nuhu contributed twice the contribution of Ade and their total contributions were ₦225,000
So the sum of their ratio is $2 + 3 + 4 = 9$.
When they made a profit of ₦135,000, based on their contributions;
Ade will receive $\frac{2}{9}$ x 135,000

$$= ₦30,000.00$$

Eyo will receive $\dfrac{3}{9} \times ₦135,000$

$$= ₦45,000.00$$

Nuhu will receive $\dfrac{4}{9} \times 135,000$

$$= ₦60,000.00$$

(b) 10 (3x - 2) = 7(5x - 4)
Multiplying the operations in the LHS and RHS, removing brackets we have
30x – 20 = 35x - 28
collecting like terms, we have
$$30x - 35x = -28 + 20$$
$$-5x = -8$$
Dividing by the coefficient of x, which is -5, we have for both sides
$$\dfrac{-5x}{-5} = \dfrac{-8}{-5}$$
$$\therefore x = 1\dfrac{3}{5}$$

3. If ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,…,15}
    A = {numbers less than 7}
    B = {multiples of 3} are subsets of ξ
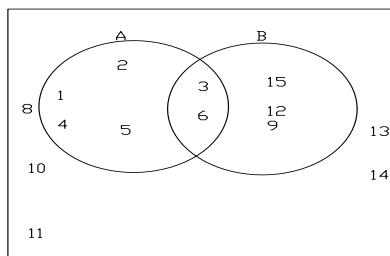(a) List the members of A and B
(b) Show the above sets in a Venn diagram
(c) List the elements of (i) $A^{1} \cap B$ (ii) $A \cup B^{1}$

Solution
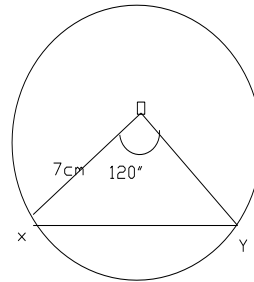(a) A = {1, 2, 3, 4, 5, 6}
    B = {3, 6, 9, 12, 15}
(b)

(c)　(i)　$A^1 = \{7, 8, 9, 10, 11, 12, 13, 14, 15\}$
　　　　$B = \{3, 6, 9, 12, 15\}$
　　　　$\therefore A^1 \cap B = \{9, 12, 15\}$

　(ii)　$B^1 = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$
　　　　$A = \{1, 2, 3, 4, 5, 6\}$
　　　　$\therefore A \cup B^1 = \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 13, 14\}$

4.　A chord XY of a circle of radius 7cm, subtends an angle of 120° at the centre.
　Find the:
　(i)　length of arc XY
　(ii)　area of the major segment which XY cuts off, and
　(iii)　area of the sector XOY.

　(i)　length of arc XY $= \dfrac{120^0}{360^0} \times 2 \times \pi \times 7 = 14.66$cm

　　　$(\pi = 3.142)$

　(ii)　Area of the major segment which XY cuts off $= \dfrac{120^0}{360^0}$ x 2 π x 7^2

　　　　　$= 102.64$cm^2

　　　Area of ΔXOY
　　　$= \frac{1}{2}$ x 7 x 7 x sin 120^0
　　　$= \frac{1}{2}$ x 49 x 0.8660
　　　$= 21.22$cm^2
　　　Therefore area of the major segment which XY cuts off
　　　$= 102.64$cm$^2 + 21.22$cm^2
　　　$= 123.86$cm^2

(iii)   Area of the sector XOY
$$= \frac{120}{360} \times \pi \times (7cm)^2$$
$$= 51.32cm^2$$

5.   The scores in an ICT Quiz is given below

Score (x)	2	3	4	5	6	7	8	9	10
Frequency(f)	2	2	4	7	8	3	4	5	3

Calculate:
(i)   total number of students
(ii)   mean scores
(iii)   median scores, and
(iv)   modal scores

Solution
(i)   Total number of students
        $2 + 2 + 4 + 7 + 8 + 3 + 4 + 5 + 3 = 38$

(ii)   Mean $= \frac{\Sigma fx}{N}$ or $\frac{\Sigma fx}{\Sigma f}$
        $\Sigma f = N = 38$
        $\Sigma fx = (2 \times 2) + (3 \times 2) + (4 \times 4) + (5 \times 7) + (6 \times 8) + (7 \times 3) + (8 \times 4) +$
        $(9 \times 5) + (10 \times 3) = 237$
        $\therefore$ mean $= \frac{237}{38} = 6.24$

(iii)   Median $= \frac{6+6}{2}$
        $= 6$

(iv)   Modal scores 6

6.(a) After examining 300 defective items, a factory quality controller came up with the
       following report. Defects in finishing 90, defects in hardness 150 and defects in
       dimension 159. Defects in hardness and finishing 30, defects in both finishing and
       dimension 24 and defects in both hardness and dimension 60. All three defects 15.
       (i)   Use a Venn diagram to illustrate this report
       (ii)   Find how many items have only one defect.
       (iii)   Find how many items have only two defects.
       (iv)   Calculate the probability of items with only two defects.

   (a) If $213_n = 117_{seven}$, find n.

<u>Solution</u>
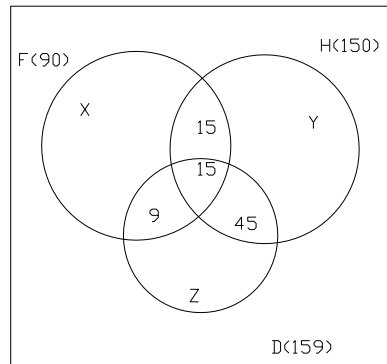
There are three (3) intersecting circles. That is $F \cap H \cap D = 15$

    (ii)   F: $x + 9 + 15 + 15 = 90$

            $\therefore x = 51$

        H: $y + 15 + 15 + 45 = 150$

            $\therefore y = 75$

        D: $z + 9 + 15 + 45 = 159$

            $\therefore \quad z = 90$

    (iii)   The only two defects are $9 + 15 + 45 = 69$

    (iv)   The probability of only two defects will be

$$= \frac{69}{300}$$

$$= \frac{23}{100}$$

$$= 0.23$$

6. $213_n = 117_{seven}$

        $2 \times n^2 + 1 \times n^1 + 3 \times n^0 = 1 \times 7^2 + 1 \times 7^1 + 7 \times 7^o$

        $2n^2 + n + 3 \qquad\qquad = 49 + 7 + 7$

        $2n^2 + n + 3 \qquad\qquad = 63$

        $\therefore 2n^2 + n - 60 \qquad\quad = 0$

        Therefore n = -5.483 or 4.983

        $\therefore n = 5$

7(a)   The mass M of a sheet of metal varies jointly with its area A and its thickness
        T. If a sheet of metal of area 250cm^2 and thickness of 1mm has a mass of
        200g:
        (i)   Find the formula which connects M, A and T
        (ii)   From the formula in (i) make A the subject of the formula
        (iii)   Hence find A when M = 960g and T = 3mm
 (b)   A right pyramid of height 15cm stands on a square base of sides 12cm.
        Calculate the volume.

   Solution
        (i)   Area of the metal sheet is length x width x thickness
                  mass of the metal steel 200g = k x 250cm^2 x 1mm
                 $\therefore$ 200 = k x 250 x 1
                 then k = 4/5
                 Therefore M = KAT
                 and this implies M = 4/5 AT
                 ALITER
                 200 = K x 250 x 0.1
                 $\therefore$ K = 8
                 and M = 8AT
        (ii)   M = 4/5 AT $\Rightarrow$ 5M = 4AT
                 $\therefore$ A = $\underline{5M}$
                           4T
        (iii)   A = 5 x $\underline{960}$ x 3
                            4
                 $\therefore$ A = 400cm^2
                 ALITER
                 M = 8AT $\Rightarrow$ A = $\underline{M}$
                                        8T
                 $\therefore$ A = $\underline{960}$
                          8 x 0.3
                      = 400cm^2
(b)   Base area = l x b = 12cm x 12cm = 144cm^2
        Volume = 1/3 x (12cm)^2 x 15cm
                = 720cm^3

8(a) Copy and complete the table for $y = 2x^2 - 3 - 6$

x	-3	-2	-1	0	1	2	3	4	5
y	18	8			-7				29
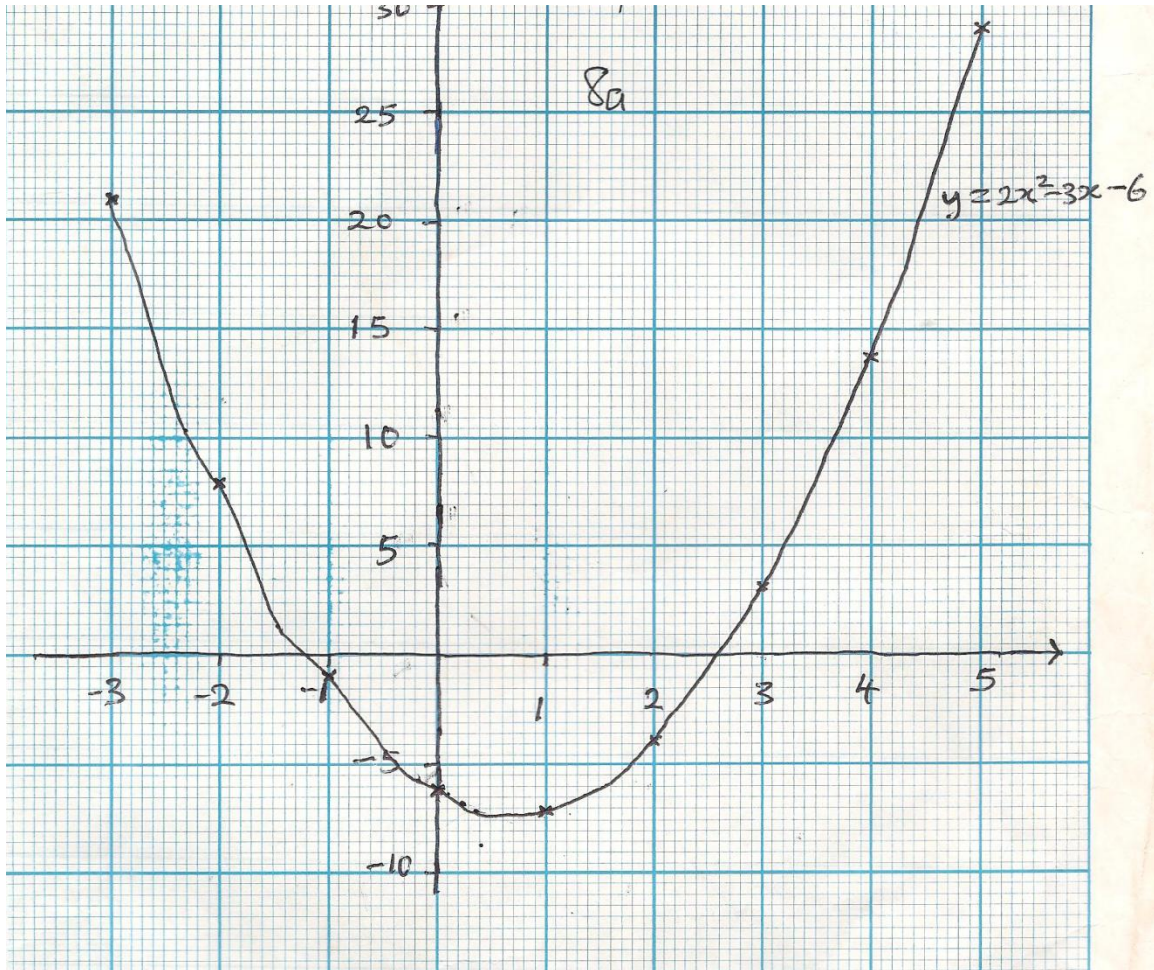
b) (i) Use your table to plot the graph of $y = 2x^2 - 3x - 6$. Use 2cm to 1unit on the x-axis
        and 2cm to 5units on the y-axis.
   (ii)   On the same axes, plot the graph of $2y - 5x + 10 = 0$
 (b)   Use your graphs to solve
       (i) $2x^2 - 3x - 6 = 0$
       (ii) $2x^2 - 3x - 6 = \frac{1}{2}(5x - 10)$
       (iii)   Find the range of values of x for which $y < 0$.

(a)

x	-3	-2	-1	0	1	2	3	4	5
y	21	8	-1	-6	-7	-4	3	14	29

(b)(i)
(ii)

(c) (i) x = -1.1   and x = 2.6   (±0.1)
    (ii) x = 2.9   and x = -0.2 (±0.1)
    (iii) The range: -1.1 < x < 2.6
                    (± 0.1 the boundaries)

9(a) Using a ruler and a pair of compasses only construct
    (i)   a triangle XYZ with /XY/ = 8cm, < YXZ = 60^0 and < XYZ = 30^0
    (ii)   the perpendicular ZT to meet XY at T.
    (iii)   the locus l_1, of points equidistant from ZY and XY

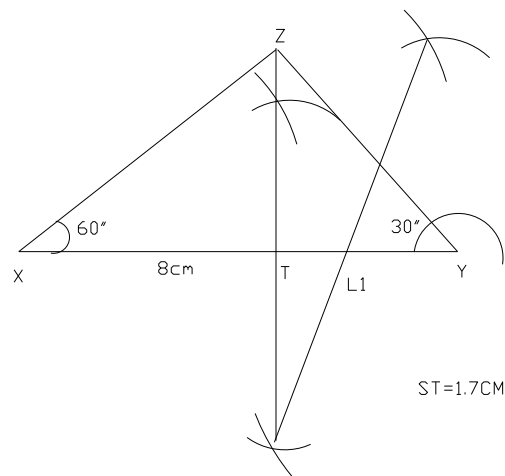(b)   If l_1 and ZT intersect at S, measure /ST/.

(c)   A fair die is rolled twice. Find the probability of:
    (i)   a sum greater or equal to 8
    (ii)   at least a four

Solution
(a)

(b)   ST = 1.7 cm ($\pm$ 0.1cm)

(c)(i)   P (sum greater than or equal to 8) = $\underline{15}$
                                                            36
              =5/12
   (ii) P (at least a four) = $\underline{11}$
                                        36

10.   An aircraft moves from a location X (28^0N, 15^0E) to another location
        Y (28^0N, 25^0W) and then to location Z(32^0S, 25^0W). The movement from X and
        Y is along the parallel of latitude and that from Y to Z is along the meridian.
        Calculate the:
        (i)   the radius of parallel of latitude 28^0N
        (ii)   distance from X to Y
        (iii)   distance from Y to Z
        (iv)   total length of the journey from X to Z.
        (v)   average speed of the aircraft if the journey takes 15 hours.
                (Take R = 6400km, π = 3.142)
        <u>Solution</u>
        (i)   r = 6400 Cos 28°
                  = 5650.56km
        (ii) $\underline{40^o}$ x 2 π 6400 x Cos 28°
                    360°
                  = $\underline{40^o}$ x 2 π 5650.56
                    360°
                  = 3945.35km

        (iii) $\underline{60^o}$ x 2 π x 6400
                360^0
                = 6702.93km

        (iv)   Total length = 3945.35km + 6702.93km
                            = 10648.28km

        (vi)   Average speed = $\underline{10648.28}$
                                        15
                                = 709.89km/hr

11.(a) In a box containing 24 identical balls, 9 are blue, 6 red, 4 are white and 5 are
        violet. If a ball is randomly selected and replaced, then a second ball is taken,
        find the probability that:
        (i)   both balls are red.
        (ii)   both are of different colours
        (iii)   they are red and a blue ball
        (iv)   both balls are violet.

(b) Find the total surface area of a cylinder of diameter 7cm and height 15cm.

Solution
a. (i)   P(both balls are red) = 6/24 x 8/24
                          = 1/16
    (ii)   P(both are of different colours) =
           1 − (1/24 x 9/24 + 6/24 x 6/24 x 4/24 x 4/24 + 5/24 x 5/24)
                = $\underline{209}$
                  288
    (iii) P(they are a red and a blue) = 6/24 x 9/24 + 9/24 x 6/24
                          = 3/16

    (iv)   P(both balls are violet) = 5/24 x 5/24
                          = $\underline{25}$
                            576
b.   Area of the two circular faces = 2 x π x $(7/2)^2$
                          = 49/2 π cm^2
        curved surface area = (2 π x 7/2 x 15)
                          = 105 π cm^2
        total surface area = ($\dfrac{49}{2}$ π + 105 π)cm^2

                = $\dfrac{259\ π}{2}$
        = 406.89cm^2 ≈ 407cm^2

12.(a) The table below is the distribution of the weekly earnings (in thousands of naira)
        of some workers in a poultry farm.

Amount in ₦1000	10	12	15	18	20
No. of Workers	8	20	6	4	2

    (a)   How many workers are on the farm?
    (b)   Calculate the mean, mode and median of their weekly earnings.

 (b)   Find the rate of 58 kobo in the naira in a property of rateable value of ₦4800.00

(c)   A trader sold an article for ₦18,500 and make a gain of 2½% . How much must
       he sell it to make a loss of 12½ %?

<u>Solution</u>

(a)　　(a) Total number of workers $= 8 + 20 + 6 + 4 + 2$
　　　　　　　　　　　　　　　$= 40$

　　　(b) Mean earning $= \dfrac{522 \times 1000}{40}$

　　　　　　　　　$= $ ₦13,050.00

　　　Median　　$= $ ₦24,000 ÷ 2

　　　　　　　　　$= $ ₦12,000.00

　　　Mode　　　$= $ ₦12,000.00

(b)　　Kobo rate: ₦ $\dfrac{58 \times 48000}{100}$

　　　　　　　$= $ ₦2784.00

(c)　　2½ % of 18,500　　$\Rightarrow \dfrac{102.5x}{100} = 18,500$

　　　　　　　　　$= 18,048.78$

　　　　　　　　$\therefore$ Selling price $= \dfrac{87.5 \times 18048.78}{100}$

　　　　　　　　　　　$= 15,792.68$

　　　<u>ALITER</u>
　　　For every ₦100 cost price, the selling price was ₦102.50 i.e. ₦102.50 selling price
　　　required ₦100 cost price. Then $\dfrac{18500 \times 100}{102.5}$

　　　　　　　　　$= $ ₦15792.68

13(a)　A man took a loan of ₦330,000.00 for 1 year at 15% per annum, deducted
　　　monthly in 11 equal installments, January to November 2009. Calculate the:
　　　(i)　　total interest
　　　(ii)　　monthly deduction from his salary

　(b)　A company's capital consist of 1,800,000 ordinary shares of 40 kobo each and
　　　160,000 preference shares of ₦1.50 each paying 7½ %. If it pays 8% on the
　　　ordinary shares, what is the profit for the year?

　<u>Solution</u>

　(a)(i) Total interest $\dfrac{15}{100} \times 330,000$

　　　　　　　$= 49,500.00$

　　(ii) Total amount $= $ ₦330,000 + ₦49,500
　　　　　　　$= 379,500$
　　　monthly deduction from his salary for 11 months equally will be ₦$\dfrac{379,500}{11}$

$= ₦34,500.00$

(b) Nominal value of the ordinary shares $= 1,800,000 \times 40$ kobo
$$= ₦720,000$$
Profit on ordinary share $= \dfrac{8}{100} \times 720,00$
$$= ₦57,600$$

Nominal value of the preference share $= ₦160,000 \times 1.50 = ₦240,00$
Profit on the preference share $= \dfrac{15}{200} \times 240,000$
$$= ₦18,000$$

$\therefore$ Total profit for the year $= ₦57,600 + ₦18,000$
$$= ₦75,600.00$$

14.(a) In a bankruptcy a creditor with a claim of ₦80,000.00 was paid ₦65,000.00. How much to the nearest naira will a creditor for ₦200,000.00 be paid?

(b) The compound interest on a sum of money invested at 12% for 3 years was ₦40,380.00. How much was invested?

(c) Aki and Pawpaw own a shop. The ratio of Aki's share to Pawpaw's is 3:2. Later pawpaw sells 2/3 of his shares to Aki for ₦82,000,000. Find the value of the shop.

Solution
(a) Dividend $= \dfrac{65000}{80000} \times 100$
$$= 81.25 \text{ kobo in the naira}$$
Therefore creditor for ₦200, 00 $= \dfrac{200,000 \times 81.25}{100}$

$$= ₦162,500.00$$

(b) Compound interest: $A = P + 40380$
$$P + 40380 = P\left(1 + \dfrac{12}{100}\right)^3$$
$$P + 40380 = (1.404928)P$$
$$0.404928P = 40380$$

$\therefore$ Principal amount invested $= ₦99721.34$

(c) Pawpaw's share $= \dfrac{8200 \times 5}{2}$

$$= ₦205,000$$

Therefore the value of the shop will be $\dfrac{205,000 \times 5}{3}$

$$= ₦512,500.00$$

15.(a) Madu bought 5000 8% preference shares nominal value ₦1.80 at ₦2.10 each.
    (i) How much did he pay for the shares?
    (ii) What is his annual income if he pays a tax at 20%.

  (b) A sales agent is paid ₦10,000 per month and in addition 7½ % commission on all sales. If in a particular year, he sold goods worth ₦1,500,000. Calculate his:
    (i) annual income before tax, and
    (ii) average monthly pay when a tax of 15% on his annual salary is made.

Solution

(a)   (i) Amount paid for the shares $= ₦2.10 \times 5,000$

$$= ₦10,500$$

    (ii) Dividend on the shares $= \dfrac{1.80 \times 5000 \times 8}{100}$

$$= ₦720.00$$

    $\text{Tax} = \dfrac{21 \times 720}{100}$

$$= ₦144$$

    Income $= ₦720 - 144 = ₦576.00$

(b)   (i) Commission received $\dfrac{7.5 \times 1,500,000}{100}$

$$= 112,500$$

    Total income before tax $= ₦120,000 + ₦112,500$

$$= ₦232,500$$

    (ii) Annual salary: $₦10,000 \times 12 = ₦120,000$

    Annual tax: $\dfrac{15}{100} \times 120,000 = ₦18,000$

    Total annual salary $= ₦232,500 - ₦18,000$

$$= ₦214,500$$

    Average monthly pay: $\dfrac{₦214,500}{12}$

$$₦17875.00$$